TWO MODES OF OPERATION OF A MODEL COMBUSTION CHAMBER AS A THERMOACOUSTIC AUTOOSCILLATING SYSTEM

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It is shown that soft and hard modes of operation of a model combustion chamber as an autooscillating system are possible. In the case of oscillations with transverse acoustic waves we: a) determined the ranges of these modes experimentally; b) detected oscillatory hysteresis ("persistence") effects and observed the abrupt appearance and disappearance of oscillations during gradual variation of the parameters. We also noted excitation of autooscillations when finite perturbations acted on the gas column in the combustion chamber in the case of the hard mode.

One of the important problems which arise in the study of vibrational combustion with acoustic oscillations in the combustion chambers of various devices (engines, stokers, furnaces, etc.) is that of determining the feedback mechanism whereby the gas column oscillating

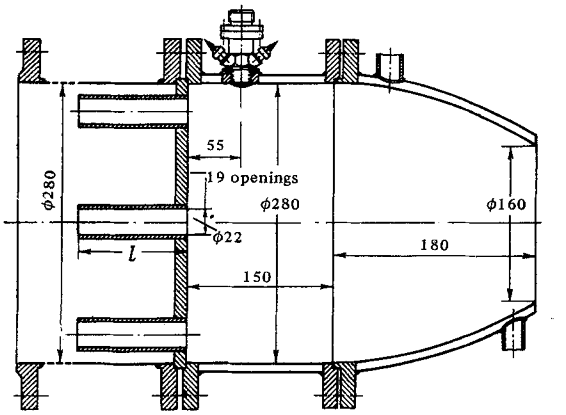

Fig. 1

in the chamber acts on the combustion zone. Because of the complexity of the process in industrial combustion chambers it is useful to carry out the appropriate studies on model chambers which make it possible to study the effects of individual stages of the process on the stability of the relatively acoustic oscillations (sometimes called the "high-frequency" oscillations).

One of the authors of the present paper proposed that model combustion chambers operating on a homogeneous mixture at low pressure be used to investigate vibrational combustion in a turbulent stream. The use of a homogeneous mixture makes it possible to exclude the effects of such processes as atomization, vaporization, and mixing of the components, and therefore to determine the role of the intrinsic combustion process in the mechanism of the phenomenon under investigation. (Paper [1], whose authors used a similar procedure, appeared at approximately the same time.)

Figure 1 is a cross section of a typical model chamber. The homogeneous combustible mixture is fed into the chamber through ducts mounted in the chamber bottom and forms turbulent flames as it burns. The chamber is water-cooled.

Determination of several factors (periodicity of vortex formation in the flame during vibrational combustion, the decisive effect of the velocity at which the fuel mixture emerges from the ducts on the stability of the combustion process, the strong damping effect of suppressors in the form of barriers only when these are placed at the root of the flame, etc.) enabled us to draw the following conclusion concerning the character of interaction of the oscillating gas column and the combustion zone. The effect of the oscillating gas on the flame during oscillatory combustion is similar in nature to the effect of sonic oscillations from an external source on a so-called "sensitive" flame (jet) [2, 3]. In other words, the nature of vortex formation in the flame during vibrational combustion is the same as that of vortex formation in open diffuse flames and in laminar and turbulent jets subjected to the action of sonic vibrations (we have in mind the hydrodynamic instability of jet flow under acoustic stimulation) [2–4].

This was used as a basis for constructing a picture of the vibrational combustion mechanism whereby feedback between the oscillating gas column and the combustion zone is maintained through the action of the oscillatory motion of the gas on the root portion of the flame. In certain hydrodynamically unstable modes of gas jet flow this action reinforces vortex formation considerably. Combined with oscillation of the gas pressure in the chamber, this renders the heat release periodic.

According to [5, 6] acoustic oscillations whose intensity is negligibly small as compared with the jet power have the ability to increase vortex intensity substantially; however, this increase is observed only if the frequency of this external perturbing force is equal to the frequency of any perturbation building up in the jet. This implies directly the first necessary condition for vibrational combustion, which consists in the fact that the (one or more) proper oscillatory frequencies of the chamber cavity must belong to the range of frequencies of the rising jet perturbations.

It should be noted that burning of a homogeneous mixture can involve feedback mechanisms occasioned by the dependence of the velocity of flame propagation on the temperature and pressure of the mixture, by periodic interruption of mixture ignition, by instability of the flame front, etc. There are grounds to suppose, however, that the action of these oscillation-stimulating factors alone cannot ensure fulfillment of the self-excitation condition for the values of the gas flow velocity and dissipative forces in the chamber. Hence, in order to emphasize the role of hydrodynamic jet instability, we assume that a single feedback channel, namely vortex formation, operates in our system.

Several authors (e.g., see [6, 7]) have established experimentally that the dependence of vortex intensity on the magnitude of the external perturbation is nonlinear. It is important to note that the character of this nonlinearity depends on the gas escape velocity and on the relationship between the natural vortex formation frequencies and the frequency of the external perturbation. This observation led us to suppose that a model chamber which constitutes a thermoacoustic autooscillating system can operate in both soft and hard modes. As we know, the latter is characterized by oscillatory hysteresis ("persistence") effects, abrupt appearance and disappearance of oscillation during gradual variation of a parameter, and excitation of autooscillations under the action of external perturbations. Hysteresis during combustion was noted in [8, 9]; the appearance of autooscillations in the combustion chamber under the action of pulses is reported by the authors of [8, 10–12]. However, these papers do not contain analyses of the effects reported. Hard appearance of oscillations in certain hydrodynamic phenomena is considered in [13].

1. The possibility of existence of soft and hard modes in a model chamber operating as an autooscillating system in the case of an arbitrary standing oscillation mode can be demonstrated schematically.

To do this we make use of a wave equation which takes account of both the generation and the dissipation of acoustic energy. Burning of a homogeneous gas mixture in a chamber takes the form of individual flames behind each duct of the head; the space between the flames, which occupy a small fraction of the chamber volume, is filled with the combustion products. This enables us to assume approximately that the chamber cavity is filled with combustion products at a single temperature and that the speed of sound is the same throughout the volume. Under this assumption the wave equation is of the form

$$\frac{\partial^2 p}{\partial t^2} - a^2 \triangle p = \frac{a^2}{c_p T}\frac{\partial Q}{\partial t} - a^2 \sigma \frac{\partial p}{\partial t}. \qquad (1)$$

Here p is the sound pressure, a is the speed of sound, c is the specific heat, T is the temperature of the combustion products, Q is the oscillatory component of the rate of heat release per unit volume, and σ is the damping factor per unit volume of the combustion chamber.

We assume that the boundary conditions are linear and inhomogeneous, but will refrain from indicating their form in the present paper.

Despite the fact that vortices arise in the boundary layer of the jet due to the action on the jet of the oscillatory velocity of the particles and of the oscillatory pressure, by virtue of the unambiguous relationship between the gas oscillation velocity and the pressure in the sound wave, the variable rate of heat release Q in the case of a homogeneous mixture flowing from the round holes in the head can be represented as a function of the variable pressure component alone:

$$Q = f(p_\tau)\,\Phi(r, \varphi, z). \qquad (2)$$

The subscript τ in this expression denotes the phase shift which can exist between the oscillations of the heat release rate and of the gas pressure in the combustion chamber. The positive function of the coordinates Φ, which is equal either to 1 or to 0, defines the region inside the combustion chamber where variable heat release occurs. The variable component of the heat release rate Q depends markedly on the intensity of the vortices being formed. Bearing in mind the nonlinear character of the dependence of the vortex intensity and the sound pressure, we assume that the function $f(p_\tau)$ is of the form

$$f(p_\tau) = \varkappa_1 p_\tau + \varkappa_2 p_\tau^2 + \varkappa_3 p_\tau^3 + \varkappa_4 p_\tau^4 - \varkappa_5 p_\tau^5, \qquad (3)$$

where the coefficients $\varkappa_n$ depend on the character of the hydrodynamic stability of the jet, i.e., where they are functions of the gas escape velocity from the head ducts, of the air-excess factor, of the frequency of gas oscillation in the chamber, etc. The coefficient $\varkappa_5$ must be positive in order for stable autooscillations to exist.

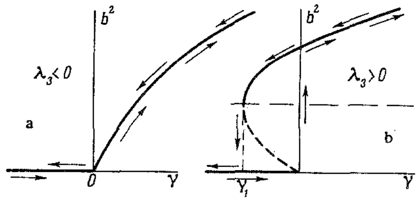

Fig. 2

Let us determine the amplitude of oscillations in the system described by Eq. (1) with the Q given by relations (2) and (3). The solution of this problem corresponding to an arbitrary standing oscillatory mode is of the form

$$p = \Psi(r, \varphi, z)\,\theta(t). \qquad (4)$$

Here $\theta(t)$ is a function of time which must be determined. It describes the dependence of the pressure amplitude on time and determines the oscillation frequency ω. The function $\Psi(r, \varphi, z)$ is a real coordinate function for the analogous particular solution

$$p = \Psi(r, \varphi, z)\, e^{i\Omega_1 t} \qquad (\Omega_1 = \omega_1 + i\delta_1)$$

of the inhomogeneous linear wave equation with given boundary conditions. Thus, expression (4) already satisfies all the boundary conditions.

Let us investigate the variation of the oscillation amplitude with time, i.e., let us find the solution for $\theta(t)$. To do this we obtain an ordinary differential equation in θ with constant coefficients: we substitute expression (4) into Eq. (1) with Q given by expressions (2) and (3), multiply the resulting equation by the coordinate function, and integrate over the entire volume. This yields the equation with real coefficients

$$\frac{d^2\theta}{dt^2} + \omega_0^2\theta = (\lambda_1 + \lambda_2\theta_\tau + \lambda_3\theta_\tau^2 + \lambda_4\theta_\tau^3 - \lambda_5\theta_\tau^4)\frac{d\theta_\tau}{dt} - k\frac{d\theta}{dt},$$

$$\lambda_n = \frac{na^2}{c_p T}\frac{\varkappa_n}{J}\int \Phi \Psi^{n+1} dv \qquad (n = 1, \ldots, 5), \qquad k = a^2\sigma$$

$$\omega_0^2 = -\frac{a^2}{J}\int \Psi \triangle \Psi dv, \quad J = \int \Psi^2 dv. \qquad (5)$$

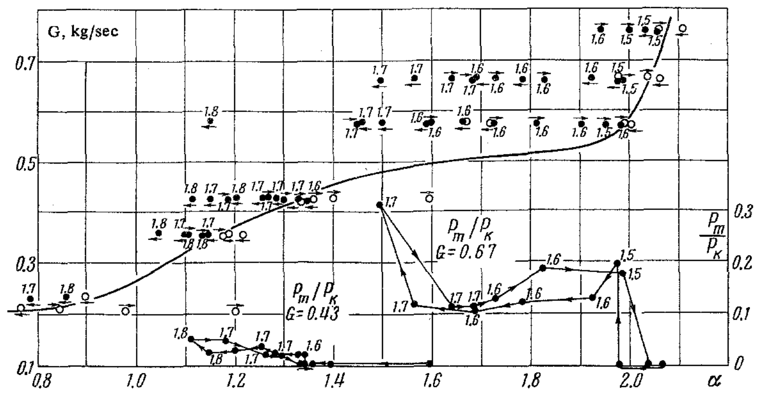

Fig. 3

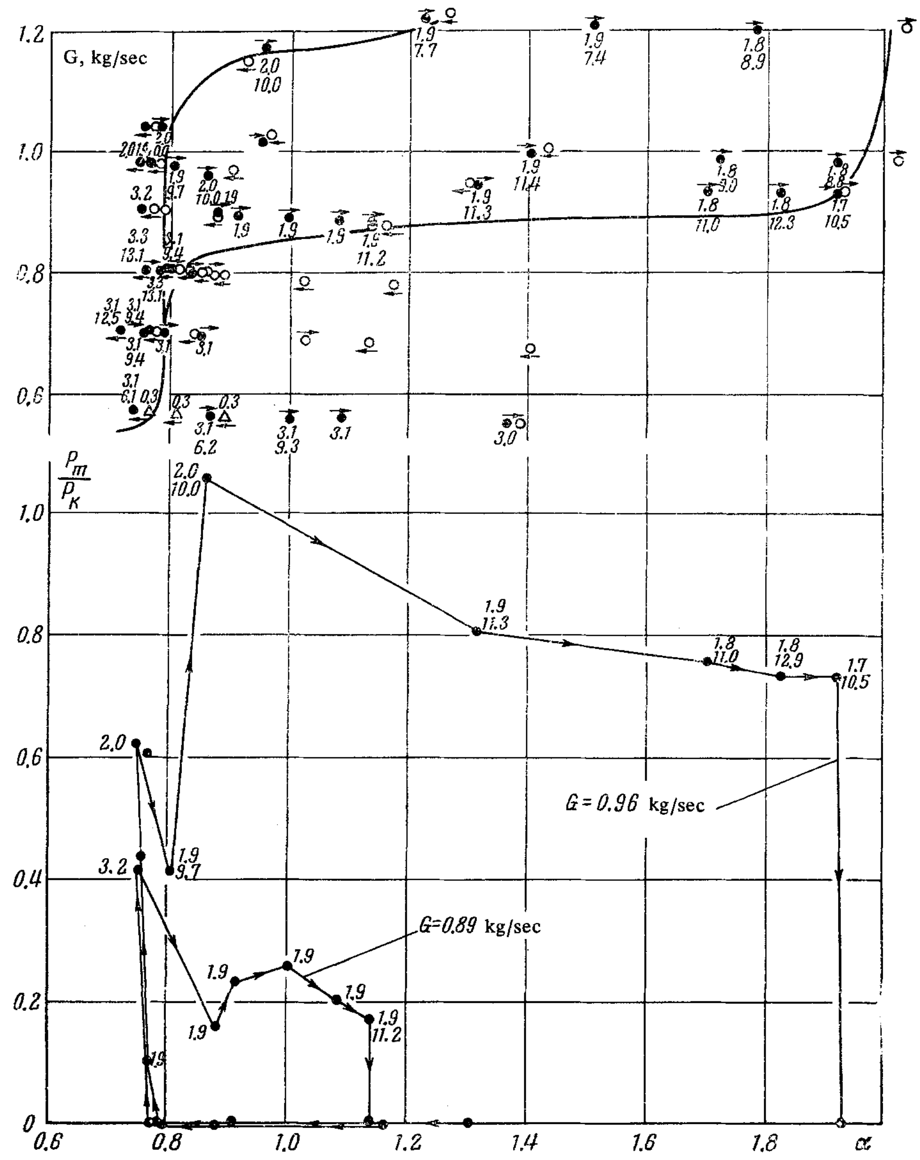

Fig. 4

Since the damping and the oscillation-exciting force are both small as compared with the elastic forces, the character of autooscillations of the gas is close to the behavior of a conservative system, and, according to the Van der Pol equation, the solution of Eq. (5) is of the form

$$\theta = b\,(t)\sin\omega t\,.$$

The amplitude b(t), which is a slowly varying function of time, is given by the first-approximation equation*

$$\frac{db}{dt} = \left(\frac{\lambda_1 b}{2} + \frac{\lambda_3 b^3}{8} - \frac{\lambda_5 b^5}{16}\right)\cos\omega_1\tau - \frac{kb}{2}\,.$$

This yields the following expression defining the amplitude in the steady state:

$$\frac{b^4}{8} - \frac{\lambda_3 b^2}{\lambda_5 4} - \frac{\lambda_1\cos\omega_1\tau - k}{\lambda_5\cos\omega_1\tau} = 0.$$

A similar equation occurs in the theory of vacuum-tube oscillators. A graphical method for solving this equation is described in [14] and is illustrated in Fig. 2, where the abscissas represent the excitation parameter

$$\gamma = \frac{\lambda_1\cos\omega_1\tau - k}{\lambda_5\cos\omega_1\tau}$$

and the ordinates represent the square of the amplitude.

For $\lambda_3 < 0$ the steady oscillations, which exist only when $\lambda_1\cos\omega_1\tau - k > 0$, are stable. With changes in system parameters (e.g., the gas escape velocity, the air excess factor) the excitation of oscillations in this case is soft. The picture is different when $\lambda_3 > 0$. Here the only stable modes are those with amplitudes $b_i > (\lambda_3/\lambda_5)^{1/2}$. For $\gamma < \gamma_1$ there exists a single stable state, namely the equilibrium state. For $\gamma > 0$ (i.e., when $\lambda_1\cos\omega_1\tau - k > 0$) the equilibrium state is unstable, the system operates in the soft mode, and autooscillations become steady under all initial conditions.

Finally, for $\gamma_1 < \gamma < 0$ the system operates in the hard mode. Depending on the initial conditions, either the equilibrium state or oscillations of amplitude $b_i > (\lambda_3/\lambda_5)^{1/2}$ can develop. Excitation of the system in this mode requires an initial "push" of an amplitude larger than that for the unstable steady state (the dashed curve in Fig. 2b). Observation of the autooscillation amplitude for the system in the hard mode during continuous and gradual variation of the parameter γ reveals that in contrast to the case of soft establishment of a steady state, oscillations appear and vanish for various values of the excitation parameters γ and for differing final amplitudes.

Depending on the sign of the coefficient λ_3 and on the values of the operating parameters, the system can operate in either the soft or the hard mode.

Since the character of the dependence of the vortex intensity on the external perturbation is determined by the hydrodynamic parameters of the jet flow and on the acoustic properties of the chamber, the sign of the coefficient λ_3 and the mode of the autooscillating system depends on the velocity with which the gas escapes from the ducts of the combustion chamber head.

2. This hypothesis was verified experimentally with the model chamber shown in Fig. 1. The chamber was fueled with a benzene-air mixture heated to 473° K. The products of combustion emerged from the nozzle at a precritical velocity. In order to increase the range of mixture escape velocities from the head ducts at which the self-excitation condition $\lambda_1\cos\omega_1\tau - k > 0$ was fulfilled and to obtain the broadest possible range of values of the air-excess coefficient α for which the chamber operated in the hard mode, we varied the oscillatory energy loss by varying the energy efflux through the chamber head. To this end we chose two heads on the basis of the results of preliminary experiments: one with ducts of length l = 100 mm and another with ducts of length l = 150 mm (see Fig. 1). These experiments indicated that in the case of ducts of length 150 mm the oscillations of the first tangential modes arose at the self-excitation boundary (usually called the vibrational burning boundary) at the minimal rate of mixture flow through the chamber head; in the case of the 100-mm ducts these oscillations appeared at the maximum flow rate.

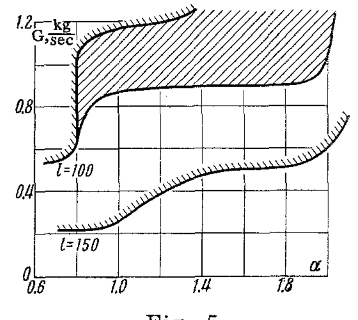

Fig. 5

The results of the principal series of experiments appear in Figs. 3–5. We determined the transition boundaries from stable to vibrational burning, and vice versa. The coordinates of the transition boundary were assumed to be those values of the mixture flow rate G and of the air-excess coefficient α at which pressure oscillations appeared or vanished in the chamber.

The pressure oscillations were recorded by means of tensometric pickups mounted along the length and circumference of the chamber. The sensitivity of the apparatus used was 0.02–0.03 kg/cm²·mm.

The upper part of Fig. 3 shows the vibrational burning boundary for the chamber with ducts of length 150 mm. The unshaded circles indicate the operating modes in which there were no oscillations. The vibrational burning region lies above the solid curve; the stable burning region lies below it. The position of the boundary was found by smoothly decreasing α from large values for various fixed air intake rates. The numbers next to the experimental points indicate the oscillation frequencies in kc. The arrows indicate the direction of change of the air-excess coefficient α. We see that the boundary at which the oscillations disappear with reverse variation of α coincides with the vibrational burning boundary for mixture flow rates through the head close to G = 0.43 kg/sec which corresponds to an (escape velocity of the mixture from the holes of U = 82 m/sec).

In the lower part of Fig. 3 we see the relative amplitude p_m/p_k (right-hand scale; p_k is the pressure in the chamber) as a function of α for two mixture flow rates through the same head. For G = 0.43 kg/sec the continuous decrease in α with passage through the value $\alpha \approx 1.35$ gave rise to oscillations whose amplitude increased continuously beginning practically from zero. With the reverse variation of α the amplitude decreased smoothly to zero, and combustion became stable. This character of appearance and disappearance of oscillations corresponds to the soft mode of the system shown in Fig. 2a. At the higher flow rate G = 0.67 kg/sec, i.e., at the higher escape velocity U ≈ 130 m/sec, continuous and gradual reduction of α immediately produced oscillations of the final amplitude at the vibrational burning boundary. Cessation of vibrational burning with the reverse variation (increase) in α also occurred at the final amplitude. This pattern of appearance of oscillations is characteristic of the hard mode of system operation shown in Fig. 2b. We must note that because of the relatively low escape velocities of the mixture under the conditions corresponding to the vibrational burning boundary, and partly because of low radiation of acoustic energy in the oscillations of the first tangential mode through the head with 150-mm ducts, the region of the hard operating mode with respect to α turned out to be very indistinct.

The upper part of Fig. 4 shows the boundary of vibrational burning (i.e., of self-excitation or of the soft mode) and the boundary of disappearance of oscillations for the chamber with the head with 100-mm

*The autooscillation frequency ω can be determined from the equation

$$\omega^2 - \omega_0^2 + \omega_1\left(\lambda_1 + {}^1/_4\,\lambda_3 b^3 - {}^1/_8\,\lambda_5 b^4\right)\sin\omega_1\tau = 0\,.$$

ducts.* As compared with the above case, the vibrational burning boundary shifted markedly toward higher mixture flow rates and (in its lower part) toward richer mixtures. For G > 0.9 kg/sec oscillations of the fundamental mode were excited on the vibrational burning boundary. These oscillations endured over a broad range of variation of the air-excess coefficient α up to the boundary of transition from vibrational to stable burning. The character of oscillation excitation for two mixture flow rates is shown in the lower part of Fig. 4. We see that continuous and gradual decreases in α led at $\alpha \approx 0.78$ ($G \approx 0.89$ and 0.96 kg/sec) to the immediate appearance of vibrational burning of the final amplitude. These lasted until $\alpha = 1.14$ and 1.93, when the oscillations ceased (at the final amplitude) and burning became stable. This was a typical example of the hard mode of operation of an autooscillating system.

A summary of our data for the two heads appears in Fig. 5 (without the experimental points for ease of examination). The shaded portion of the parameter plane is the hard-mode domain where hysteresis (persistence) of the vibrations was noted. The possibility of excitation of vibrational burning in a chamber during stable combustion, given a certain "push," was confirmed experimentally for these values of the operating parameters G and α. The perturbation source was the detonation of a quantity of black gunpowder in a percussion device placed tangentially to and at a distance of 45 mm from the face of the head. For example, for G = 1.04 kg/sec, $\alpha = 0.97$ and initially stable burning, detonation of 2 g of gunpowder in the chamber with the 100-mm ducts produced autooscillations of a frequency of ~ 2 kc (the first tangential mode); an oscillation overtone at 10 kc was also noted. The amplitude of the steady oscillations was approximately equal to the amplitude obtained with oscillatory hysteresis for these values of the parameters G and α.

The possibility of exciting oscillations by means of a finite pressure pulse in the chamber proved that the hard mode prevailed in this case and that the hysteresis effect was related to the fundamental properties of the autooscillating system under consideration. As expected, detonations of from 0.5 to 2.5 g of gunpowder in the chamber with the head with 150-mm ducts did not produce autooscillations (the modes G = 0.5 kg/sec, $\alpha = 1.9$ and G = 0.4 kg/sec, $\alpha = 1.3$).

We regard our experimental verification of the existence of soft and hard operating modes of the same combustion chamber (depending on the mixture flow rate from the head ducts) as further proof of the fundamental role of hydrodynamic instability of the flames in the feedback mechanism involved in vibrational combustion.

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*The triangles indicate modes in which low-frequency oscillations were noted.

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